B.Sc Part II
PAPER – III Notes – 10

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Functions continuous on closed intervals.
Properties of functions which are continuous on closed intervals.

Th. If a function is continuous in a closed interval, then it is bounded therein

Proof Let f be a function defined and continuous in a closed interval I.
Show that if the function f is not bounded then it ~~fall~~ fails to be continuous at some point of the closed interval I.

Let if possible, f be not bounded above, so that for each positive integer n $\exists$ a point $x_n \in I$ such that $f(x_n) > n$.
Now $[x_n]$, being a sequence in the closed interval I, is bounded and has at least one limit point say ξ.
A closed interval is a closed set

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and so $\xi \in I$.
Further since ξ is a limit point of the sequence $[x_n]$, therefore there exists a subsequence $\{x_{n_k}\}$ of $\{x_n\}$ such that $x_{n_K} \to \xi$

as $K \to \infty$
Also since $f\{x_{n_K}\} > n_K \;\forall K$

therefore the sequence $|f(x_{n_K})|$ diverges to ∞

Thus $\exists$ a point ξ of I such that a sequence $\{x_{n_K}\}$ in I converges to ξ but

$$\lim_{K \to \infty} f(x_{n_K}) \neq f(\xi)$$

Thus f is not continuous at ξ, which is a contradiction and hence the function is bdd above.
By considering a function f, it can be ~~above~~ shown in a similar way that the fun. f is also bdd below. Hence the function is bdd.